HORNSBY GIRLS' HIGH SCHOOL

2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- **Reading Time- 5 minutes**
- **Working Time – 2 hours**
- **Write using a black or blue pen**
- **Approved calculators may be used**
- **A table of standard integrals is provided at the back of this paper.**
- **All necessary working should be shown for every question.**
- **Begin each question on a fresh sheet of paper.**

Total marks (84)

- **Attempt Questions 1-7**
- **All questions are of equal value**

Total Marks – 84
Attempt Questions 1-7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Let A (3,4) and B (-2,5) be points in the plane. Find the co-ordinates of the Point C which divides the interval externally in the ratio 1:3. **1**

(b) A committee of 4 boys and 4 girls is chosen from a class of 8 boys and 6 girls

(i) Calculate the probability that the committee includes the eldest boy and excludes the eldest girl **2**

(ii) Find the number of ways the committee of 4 boys and 4 girls can be arranged in a circle so that each of the girls are separated. **1**

(c) For the function $f(x) = \frac{1}{2}\cos^{-1}\frac{1}{3}x$

(i) State the range and the domain of $f(x)$ **2**

(ii) Sketch the graph of $y = f(x)$ **1**

(d) Using the expansion of $sin(A-B)$ or otherwise prove the exact value of $sin\,\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$ **2**

(e) The curves $y = log_e x$ and $y = -x^2 + 1$ intersect at the point P (1,0). Find the acute angle between the tangents to the curves at the point P. Give your answer to the nearest degree. **3**

Question 2 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Solve $|x-1| \le |x+1|$ **2**

(b) Express cos ($2\sin^{-1}\frac{a}{b}$) in terms of a and b. **2**

(c) The graph of $y = \sin x$ for $\pi/_{12} \le x \le \pi$ is rotated about the x axis. Calculate the volume of the solid generated. **3**

(d) Write down the general solution of the equation
$\sqrt{3}\cos 2x - \sin 2x = 2$ **3**

(e) In how many ways can the letters of the word GEOLOGIST be arranged so that the letters G will be together? **2**

Question 3 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) Calculate $\int_0^{\sqrt{\frac{27}{2}}} \frac{1}{9+2x^2}\,dx$ **3**

(b) Calculate the area between the curve $y = \sin^{-1}x$, *the* x axis and the line $x=1$ **2**

(c) Prove $\frac{\sin 2\theta + \sin\theta}{1+\cos 2\theta + \cos\theta} = \tan\theta$ **2**

(d) An eight sided die has 5 green faces and 3 blue faces. If the die is tossed 100 times find the most likely number of green faces and the probability of this occurring (*correct to 3 decimal places)* **3**

(e) Find the value of n, if the coefficients of x^5 and x^6 in the expansion of $(3 + 2x)^n$ have the same value **2**

Question 4 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) A particle travelling in a straight line is governed by the equation $v^2 = 15 + 2x - x^2$ where v is the velocity in m/s and x is the distance travelled in time t seconds.

(i) Prove that the particle undergoes simple harmonic motion **1**

(ii) (1) Find the centre of the motion **1**
(2) Find the amplitude of the motion **1**
(3) Find the period of the motion **1**

(iii) Write down the maximum speed and the maximum acceleration **2**

(iv) Given that the particle was originally at its equilibrium position write down an equation for the position $x = f(t)$ and hence or otherwise find the velocity when $t = \pi/4\ s$ **3**

(b) Prove by Mathematical Induction that:
$2(1!) + 5(2!) + 10(3!) + \ldots\ldots\ldots + (n^2 + 1)n! = n(n+1)!$
for all positive integers $n \geq 1$ **3**

Question 5 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The Polynomial $2x^3 + ax^2 + bx + 6$ has $(x - 1)$ as a factor and leaves a remainder of -12 when divided by $(x - 2)$. Find the values of a and b. **2**

(b) Solve the equation $x^3 + 2x^2 - 5x - 6 = 0$ given that one of its roots is equal to the sum of the other two roots **2**

(c) Two straight roads intersect at right angles. At a given instant a car is 30 km from the intersection and is travelling towards it at 50 km/h while the truck is 40 km from the intersection and is travelling away from it at 40 km/h.. At what rate is the direct distance between them changing at this instant? **2**

(d) Show that $\dfrac{d}{dx}\left[\sin^{-1}\left(\tfrac{1}{2}\sin x\right)\right] = \dfrac{\cos x}{\sqrt{4-\sin^2 x}}$

hence evaluate $\displaystyle\int_0^{\pi/2} \frac{\cos x}{\sqrt{4-\sin^2 x}}\,dx$ **3**

(e) Using the substitute $u = 1 - 3x$

evaluate $\displaystyle\int_0^{\frac{1}{3}} 3x(1-3x)^4\,dx$ **3**

Question 6 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) A rocket is fired at a speed of V m/s at an angle θ to the horizontal where $\tan\theta = \frac{4}{3}$. Neglecting air resistance and using acceleration due to gravity as $g = 10\text{m/s}^2$

(i) Show that the horizontal position x and the vertical position y of the rocket at any time t is given by
$x = \frac{3Vt}{5}$ and $y = \frac{4Vt}{5} - 5t^2$ **2**

(ii) The rocket hits a target which has the coordinates (324,27)
Find the value of V **3**

(b) Using the expansion of $(1+x)^n$

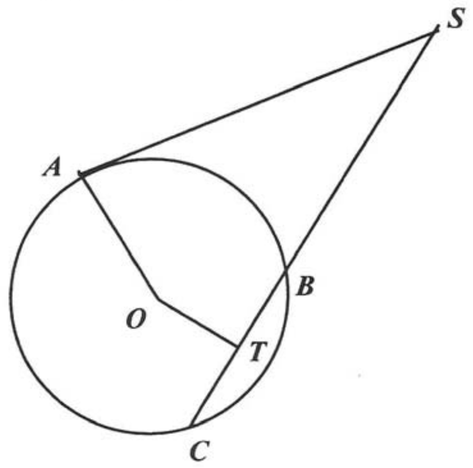
Show $\frac{-1}{n+1} = -{}^nc_0 + \frac{1}{2}({}^nc_1) - \frac{1}{3}({}^nc_2) + \text{------------} + \frac{(-1)^{n+1}}{n+1}({}^nc_n)$ **3**

(c) A, B, C are three points on the circumference of a circle, centre O. The tangent at A meets CB produced at S. T is the mid point of BC.

Prove that

(i) $TOAS$ is a cyclic quadrilateral **2**

(ii) $\angle OAT = \angle OST$ **2**



Question 7 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The tangent at $P(2ap, ap^2)$ To the parabola $x^2 = 4ay$ meets the x axis at A.

(i) Find the co-ordinates of A **2**

(ii) If S is the Focus of this parabola. Prove SA is perpendicular to AP **1**

(iii) Show that the equation of the locus of the Centre C of the circle which passes through the three points P, S and A is a parabola and write down the coordinates of its vertex **3**

(b) The rate at which a body warms or cools in air is proportional to the difference between its temperature T and the constant temperature of its surroundings S. The temperature obeys the differential equation $\frac{dT}{dt} = k(T - S)$. You may assume the solution $T = S + Ae^{kt}$

(i) A cup of boiling water at $100°$C and a cup of iced water at $0°$C are placed simultaneously in a room which has a temperature of $25°$C. After 5 minutes the temperature of the boiling water has fallen to $55°$C and the temperature of the iced water has risen to $15°$C. Find the time at which the temperature of the two liquids differs by $10°$C.
(Give your answer correct to two decimal places) **4**

(ii) Draw a graph of the behaviour of the temperature T of both liquids as t becomes large. **2**

End of Examination

Question 1 (12 marks)

$-1 \qquad 3$

(a) $A(3,4)$ and $B(-2,5)$

$x = \frac{2+9}{2} \qquad y = \frac{5+1}{2}$

$x = 5\frac{1}{2} \qquad y = 3\frac{1}{2}$

$C\left(5\frac{1}{2}, 3\frac{1}{2}\right)$ ✓

(b) 8 boys 6 girls

↓

4 boys 4 girls

(i) $n(S) = {}^8C_4 \times {}^6C_4$

$= 70 \times 15$

$= 1050$ ✓

$n(E) = {}^7C_3 \times {}^5C_4$

$= 35 \times 5$

$= 175$

$P(E) = \frac{175}{1050}$ ✓

$= \frac{1}{6}$

(ii) $3! \times 4!$

$= 144$ ✓

(c) $f(x) = \frac{1}{2}\cos^{-1}\frac{1}{3}x$

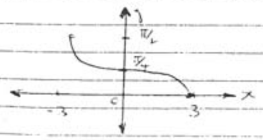
$2y = \cos^{-1}\frac{x}{3}$

Range $0 \le 2y \le \pi$

$0 \le y \le \frac{\pi}{2}$ ✓

Domain $-1 \le \frac{x}{3} \le 1$

$-3 \le x \le 3$ ✓



✓

(d) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$\sin\frac{\pi}{12} = \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\times\frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ ✓

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$ ✓

(e)

$f(x) = \ln x$	$f(x) = -x^2+1$
$f'(x) = \frac{1}{x}$	$f'(x) = -2x$
$f'(1) = 1$	$f'(1) = -2$
$m_1 = 1$	$m_2 = -2$

✓

$\tan\theta = \frac{m_1 - m_2}{1+m_1m_2}$

for acute angle θ

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

✓

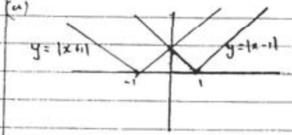
$\tan\theta = \left|\frac{1+2}{1-2}\right|$

$\tan\theta = 3$ ✓

$\theta = 72^\circ$ (Nearest degree)

Question 2 12 Marks.

(a)



✓ ✓

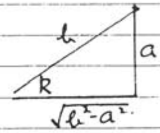
$|x-1| \le |x+1|$

$x \ge 0$

(b) $\cos\left(2\sin^{-1}\frac{a}{b}\right)$

Let $\sin^{-1}\frac{a}{b} = k$

$\sin k = \frac{a}{b}$



$\cos 2k = 2\cos^2 k - 1$ ✓

$= 2\left(\frac{b^2-a^2}{b^2}\right) - 1$

$= \frac{2b^2-2a^2-b^2}{b^2}$

$= \frac{b^2-2a^2}{b^2}$ ✓

(c) $V = \pi\int y^2\,dx$

$= \pi\int_{\pi/12}^{\pi} \sin^2 x\,dx$ ✓

$= \frac{\pi}{2}\int_{\pi/12}^{\pi} 1 - \cos 2x\,dx$ ✓

$= \frac{\pi}{2}\left[x - \frac{1}{2}\sin 2x\right]_{\pi/12}^{\pi}$ ✓

$= \frac{\pi}{2}\left[[\pi - 0] - \left[\frac{\pi}{12} - \frac{1}{4}\right]\right]$

$= \frac{\pi}{2}\left[\frac{11\pi}{12} + \frac{1}{4}\right]$

$= \frac{\pi}{2}\left[\frac{11\pi+3}{12}\right]$

$= \frac{\pi}{24}[11\pi+3]$ cubic units ✓

(d) $\sqrt{3}\cos 2x - 1\sin 2x = 2$

use subsidiary angle

$R = \sqrt{4} \qquad \tan\theta = \frac{1}{\sqrt{3}}$

$R = 2 \qquad \theta = \frac{\pi}{6}$

$\therefore\ 2\cos\left(2x+\frac{\pi}{6}\right) = 2$ ✓

$\cos\left(2x+\frac{\pi}{6}\right) = 1$

$\cos\left(2x+\frac{\pi}{6}\right) = \cos 0$

$\therefore\ 2x + \frac{\pi}{6} = 2n\pi \pm 0$

$2x = 2n\pi - \frac{\pi}{6}$ ✓

$x = n\pi - \frac{\pi}{12}$

(e) G E O L I S T ✓

G O

$\boxed{G/G}$ E O L I S T

O

$= \frac{8!}{2!}$ ✓

$= 20160$

Question 3

(a) $\displaystyle\int_0^{\sqrt{27/2}} \frac{1}{9+2x^2}\,dx$

$= \frac{1}{2}\displaystyle\int_0^{\sqrt{27/2}} \frac{1}{\frac{9}{2}+x^2}\,dx$

$= \frac{1}{2}\displaystyle\int_0^{\sqrt{27/2}} \frac{1}{\left(\frac{3}{\sqrt2}\right)^2+x^2}\,dx$

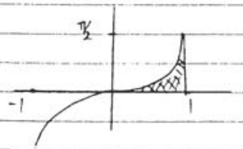
$= \frac{1}{2} \times \frac{\sqrt2}{3}\left[\tan^{-1}\frac{\sqrt2\, x}{3}\right]_0^{\sqrt{27/2}}$

$= \frac{\sqrt2}{6}\left(\left[\tan^{-1}\frac{\sqrt2 \times 3\sqrt3}{3\sqrt2} - \tan^{-1}0\right]\right)$

$= \frac{\sqrt2}{6}\left(\tan^{-1}\sqrt3\right]$

$= \frac{\sqrt2}{6} \times \frac{\pi}{3}$

$= \frac{\sqrt2\,\pi}{18}$



$A = \frac{\pi}{2} - \displaystyle\int_0^{\frac{\pi}{2}} \sin x\,dx$

$\frac{\pi}{2} - \left(\left[-\cos x\right]_0^{\frac{\pi}{2}}\right)$

$= \frac{\pi}{2} - (0+1)$

$= \frac{\pi}{2} - 1$ sq units.

(c)

LHS $= \dfrac{2\sin\theta\cos\theta + \sin\theta}{1+2\cos^2\theta - 1 + \cos\theta}$

$= \dfrac{\sin\theta\,(2\cos\theta+1)}{\cos\theta\,(2\cos\theta+1)}$

$= \tan\theta$

$=$ RHS

(d) Let $g = \frac{5}{8}$

$h = \frac{3}{8}$

$(g+h)^{100} =$

max Coef : $\dfrac{n-r+1}{r}\;\dfrac{\frac{3}{8}}{\frac{5}{8}} \geq 1$

$\dfrac{101-r}{r}\;\frac{3}{5} \geq 1$

$303 - 3r \geq 5r$

$8r \leq 303$

$r \leq 37.875$

$r = 37$

$\therefore$

MAX Coef Term $\;{}^{100}C_{37}\left(\frac{5}{8}\right)^{63}\left(\frac{3}{8}\right)^{37}$

(e) most likely green faces is 63

Probability of 63 green faces $= \binom{100}{37}\left(\frac{5}{8}\right)^{63}\left(\frac{3}{8}\right)^{37}$

$= 0.082$

Term in x^5

${}^nC_5\, 3^{n-5}\, 2^5$

Term in x^6

${}^nC_6\, 3^{n-6}\, 2^6$

${}^nC_5\, 3^{n-5}\, 2^5 = {}^nC_6\, 3^{n-6}\, 2^6$

${}^nC_5 \div {}^nC_6 = \frac{2}{3}$

$\dfrac{n!}{(n-5)!\,5!} \times \dfrac{(n-6)!\,6!}{\cancel{n!}} = \frac{2}{3}$

$\dfrac{6}{(n-5)} = \frac{2}{3}$

$2n - 10 = 18$

$2n = 28$

$n = 14$

Question 4

$v^2 = 15 + 2x - x^2$

Accel $= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(\frac{15}{2} + x - \frac{1}{2}x^2\right)$

$= 1 - x$

$= -1(x-1)$

$= -n^2(x-h)$ where $n = 1$

$h = 1$

Simple harmonic motion

Centre is at $x = 1$

(α) Centre is $x = 1$

(β) Particle stops $x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5$ and $x = -3$

$\therefore$ Amplitude $= 4$ units.

(γ) Period $T = \frac{2\pi}{n}$

$= 2\pi$ Seconds

(iii) MAX velocity is at $x = 1$

(ie) $v^2 = 15 + 2 - 1$

$v^2 = 16$

$v = \pm 4$

MAX Speed is 4 m/s.

MAX acc is at $x = 5$ or $x = -3$

(ie) acc $= -1(5-1)$ or $-1(-3-1)$

MAX acc $= \pm 4$ m/s^2

(iv) $x = 4\sin t$

$x = 4\sin t$

Vel $= \dot{x} = 4\cos t$

When $t = \frac{\pi}{4}$

vel $= 4 \times \frac{1}{\sqrt2}$

$= 2\sqrt2$ m/s

(b) When $n = 1$

LHS $= 2 \times 1!$

$= 2$

RHS $= 1(2)!$

$= 2$

$\therefore$ True For $n = 1$

Assume True For $n = k$

(ie) Assume

$\boxed{2(1!) + 5\times(2!) + \ldots + (k^2+1)k! = k(k+1)!}$

RQTP

$2(1!) + 5\times(2!) + \ldots + (k^2+1)k! + ((k+1)^2+1)(k+1)!$

$= (k+1)(k+2)!$

LHS $= k(k+1)! + ((k+1)^2+1)(k+1)!$

$= k(k+1)! + (k^2+2k+2)(k+1)!$

$= (k+1)\,[\,k\,k! + (k^2+2k+2)k!$

$= (k+1)\,k!\,[k + k^2+2k+2]$

$= (k+1)\,k!\,[k^2+3k+2]$

$= (k+1)\,k!\,(k+2)(k+1)$

$= (k+1)\,(k+2)!$

$=$ RHS

True for $n = k+1$

ince True for $n=1$ Then

True for $n = 2, 3, 4$ all $n \geq 1$

---

uestion 5

a) Let $P(x) = 2x^3 + ax^2 + bx + 6$

$P(1) = 0$

(ie) $2 + a + b + 6 = 0$

$a + b + 8 = 0 \quad — (1)$

$P(2) = -12$

(ie) $16 + 4a + 2b + 6 = -12$

$4a + 2b + 34 = 0$

$2a + b + 17 = 0 \quad — (2)$

(2) − (1) $\quad a + 9 = 0$

$a = -9$

$b = 1$

$x^3 + 2x^2 - 5x - 6 = 0$

Let Roots be $\alpha, \beta, (\alpha+\beta)$

um: $2\alpha + 2\beta = -2$

$\alpha + \beta = -1 \quad — (1)$

oduct: $\alpha\beta(\alpha+\beta) = 6 \quad — (2)$

rom (1)

$\beta = -1 - \alpha$

into (2) $\quad \alpha(-1-\alpha)(-1) = 6$

$\alpha + \alpha^2 = 6$

$\alpha^2 + \alpha - 6 = 0$

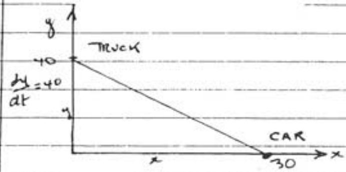
$(\alpha+3)(\alpha-2) = 0$

$\alpha = -3 \qquad \alpha = 2 \quad$ and $\alpha + \beta = -1$

$\beta = 2 \qquad \beta = -3$

oots $\quad -3, 2, -1$

(c)



-50

$\frac{dx}{dt} = -50$

(1)

$z^2 = x^2 + y^2$

$\cancel{2}z\frac{dz}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$

(2)

$\frac{dz}{dt} = \frac{30 \times -50 + 40 \times 40}{50}$

$= \frac{-1500 + 1600}{50}$

$= 2$ Km/h.

(d)

$\frac{d}{dx}\sin^{-1}\left(\frac{1}{2}\sin x\right)$

$= \frac{1}{\sqrt{1 - \frac{1}{4}\sin^2 x}} \times \frac{1}{2}\cos x$

$= \frac{\cos x}{\sqrt{4 - \sin^2 x}}$

$\therefore \int_0^{\pi/2} \frac{\cos x \, dx}{\sqrt{4 - \sin^2 x}} = \left[\sin^{-1}\left(\frac{1}{2}\sin x\right)\right]_0^{\pi/2}$

$= [\sin^{-1}(\tfrac{1}{2})] - [\sin^{-1} 0]$

$= \pi/6$

) $I = \int_0^{1/3} 3x(1-3x)^4 \, dx$

Let $u = 1 - 3x$

$du = -3dx$

$dx = -\frac{1}{3}du$

rminal Values

$x = \frac{1}{3} \Rightarrow u = 0$

$x = 0 \Rightarrow u = 1$

$I = -\frac{1}{3}\int_1^0 (1-u)u^4 \, du$

$= \frac{1}{3}\int_0^1 u^4 - u^5 \, du$

$= \frac{1}{3}\left[\frac{u^5}{5} - \frac{u^6}{6}\right]_0^1$

$= \frac{1}{3}\left[\frac{1}{5} - \frac{1}{6}\right] - [0]$

$= \frac{1}{90}$

(ii) Trajectory

$y = x\tan\theta - \frac{gx^2}{2v^2}(1 + \tan^2\theta)$ — Given

where $\tan\theta = \frac{4}{3}$

$x = \frac{3vt}{5}$ $\qquad x = 324$

$y = 27$

$t = \frac{5x}{3v}$ $\qquad g = 10$

$y = -5\left(\frac{5x}{3v}\right)^2 + \frac{4v}{5}\left(\frac{5x}{3v}\right)$

$27 = 324 \times \frac{4}{3} - 5 \times \frac{(324)^2}{v^2}\left(\frac{25}{9}\right)$ ✓

$9 \times 27 = 12 \times 324 - \frac{125 \times 324^2}{v^2}$

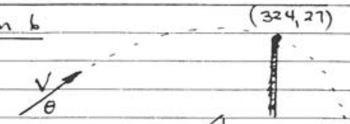
$243 = 3888 - \frac{13122000}{v^2}$

$3645v^2 = 125 \times 324^2$

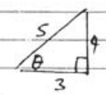
$v^2 = \frac{125 \times 324^2}{3645}$

$V = 60$ ✓ $\qquad$ 3

uestion 6



$\tan\theta = \frac{4}{3}$



Vertical	Horizontal.
$\ddot{y} = -10$	$\ddot{x} = 0$
$\dot{y} = -10t + \frac{4V}{5}$	$\dot{x} = \frac{3V}{5}$
$y = -5t^2 + \frac{4Vt}{5}$ ✓	$x = \frac{3Vt}{5}$ ✓

(2)

(b)

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_n x^n$

Integrate Both sides with respect to x

$\frac{(1+x)^{n+1}}{n+1} = {}^nC_0 x + \frac{1}{2}{}^nC_1 x^2 + \frac{1}{3}{}^nC_2 x^3 + \cdots + \frac{{}^nC_n x^{n+1}}{(n+1)} + k$ ✓

Let $x=0$ To Find k

$\frac{1}{n+1} = k$ ✓

$\therefore \frac{(1+x)^{n+1} - 1}{n+1} = {}^nC_0 x + \frac{1}{2}{}^nC_1 x^2 + \frac{1}{3}{}^nC_2 x^3 + \cdots + \frac{{}^nC_n x^{n+1}}{(n+1)}$

Let $x = -1$

✓

$\frac{-1}{n+1} = -{}^nC_0 + \frac{1}{2}{}^nC_1 - \frac{1}{3}{}^nC_2 + \cdots \frac{(-1)^{n+1}\,{}^nC_n}{(n+1)}$

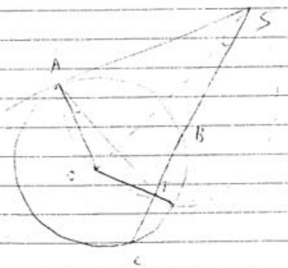
3

(i) $\angle SAO = 90^\circ$ (Tangent $\perp$ Radius)

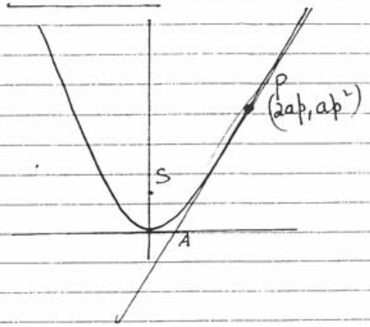
$\angle OTB = 90^\circ$ ($\because$ Line joining centre of circle to mid point of chord is 90° to chord)

$\therefore$ TOAS is a cyclic quad because opposite angles are supplementary ✓ (2)

$\angle OAT = \angle OST$ (angles at the circumference, standing on same arc $\underline{OT}$) ($\because$ TOAS cyclic quad) (2)



QUESTION 7



(i) Tangent at $(2ap, ap^2)$

$y = px - ap^2$.

Sub $y = 0$

$px = ap^2$

$x = ap$

$\therefore$ A is $(ap, 0)$ (2)

(ii) $S(0, a)$ $\quad A(ap, 0)$

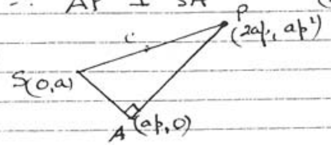
Gradient SA $= \dfrac{a}{-ap}$

$= -\dfrac{1}{p}$

Gradient AP $= p$

$\therefore AP \perp SA$ (1)

(iii)



Centre of Circle $\left(ap, \dfrac{ap^2 + a}{2}\right)$

$x = ap \qquad y = \dfrac{ap^2 + a}{2}$

$p = \dfrac{x}{a}$

$y = \dfrac{a \times \frac{x^2}{a^2} + a}{2}$

$2ay = x^2 + a^2$

$x^2 = 2ay - a^2$

$x^2 = 2a\left(y - \frac{a}{2}\right)$ Locus

This is the form of the general equation of a Parabola

$(x - h)^2 = 4a(y - k)$

$\therefore$ The Locus is the parabola

$(x - 0)^2 = 2a\left(y - \frac{a}{2}\right)$

which has vertex $\left(0, \frac{a}{2}\right)$ (3)

(i) $T = S + Ae^{kt}$

$\dfrac{dT}{dt} = kAe^{kt}$

$= k(T - S)$

You may assume this in this question

(ii) BOILING WATER

$T = 25 + Ae^{k_1 t}$

$\left.\begin{matrix} t = 0 \\ T = 100 \end{matrix}\right]$ $100 = 25 + A$

$A = 75$

$\therefore T = 25 + 75e^{k_1 t}$

$\left.\begin{matrix} T = 55 \\ t = 5 \end{matrix}\right]$ $55 = 25 + 75e^{5k_1}$

$30 = 75e^{5k_1}$

$e^{5k} = \frac{2}{5}$

$k_1 = \frac{1}{5} \ln \frac{2}{5}$

$T = 25 + 75e^{\frac{t}{5} \ln \frac{2}{5}}$ ✓

ICED WATER

$T = 25 + Ae^{k_2 t}$

$\left.\begin{matrix} t = 0 \\ T = 0 \end{matrix}\right]$ $0 = 25 + A$

$A = -25$

$T = 25 - 25e^{k_2 t}$

$\left.\begin{matrix} T = 15 \\ t = 5 \end{matrix}\right]$ $15 = 25 - 25e^{5k_2}$

$-10 = -25e^{5k_2}$

$e^{5k_2} = \frac{2}{5}$

$k_2 = \frac{1}{5} \ln \frac{2}{5}$

$T = 25 - 25e^{\frac{t}{5} \ln \frac{2}{5}}$ ✓

Difference in Temps $= 10^\circ C$

$10 = \left(25 + 75e^{\frac{t}{5} \ln \frac{2}{5}}\right) - \left(25 - 25e^{\frac{t}{5} \ln \frac{2}{5}}\right)$ ✓

$10 = 100e^{\frac{t}{5} \ln \frac{2}{5}}$

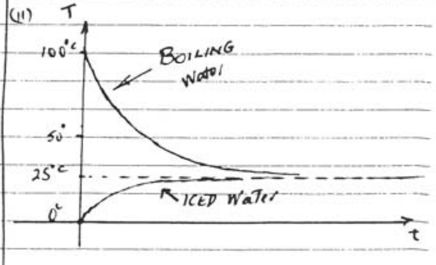
$\cdot 1 = e^{\frac{t}{5} \ln \frac{2}{5}}$

$\frac{t}{5} \ln \frac{2}{5} = \ln \cdot 1$

$t = 5 \times \dfrac{\ln \cdot 1}{\ln \frac{2}{5}}$

$t = 12.56$ mins ✓ (4)

(iii)



(2)